Arrangements of Conic Arcs

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a joint work with

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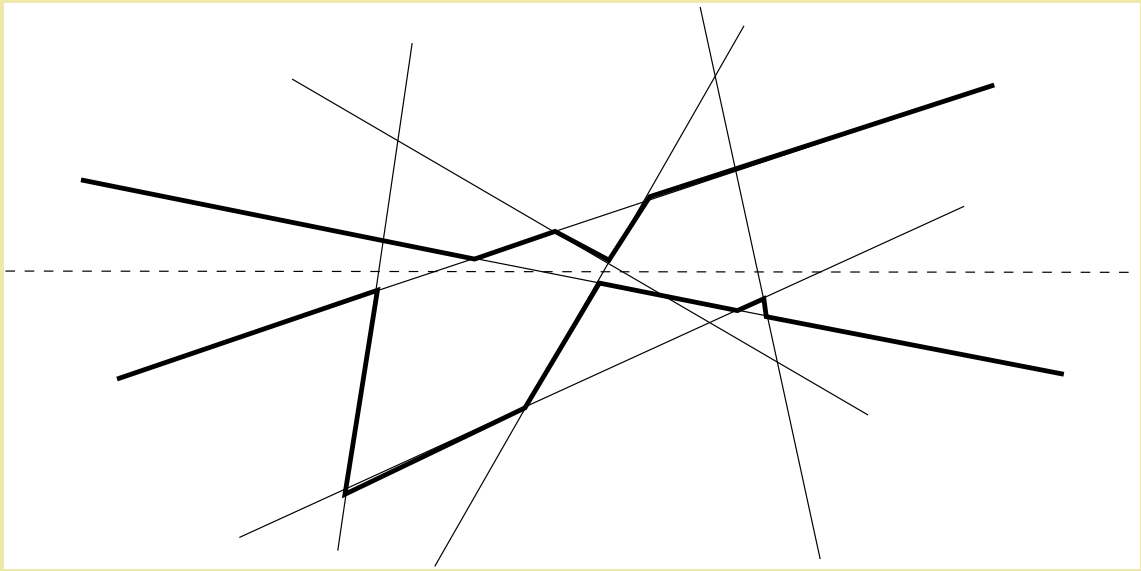
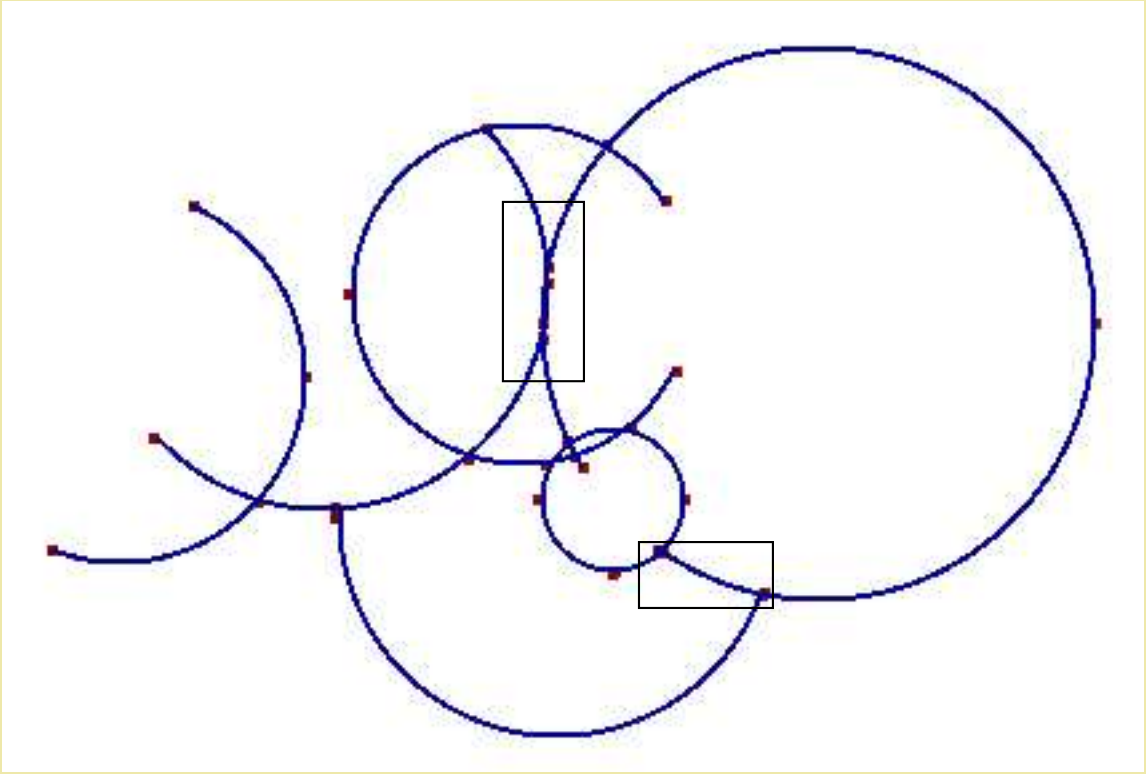
Outline

1. Arrangements Intro
2. CGAL Introduction
3. The Curved Kernel
4. The Algebraic Kernel
5. The Arrangement Traits

Arrangement of Lines

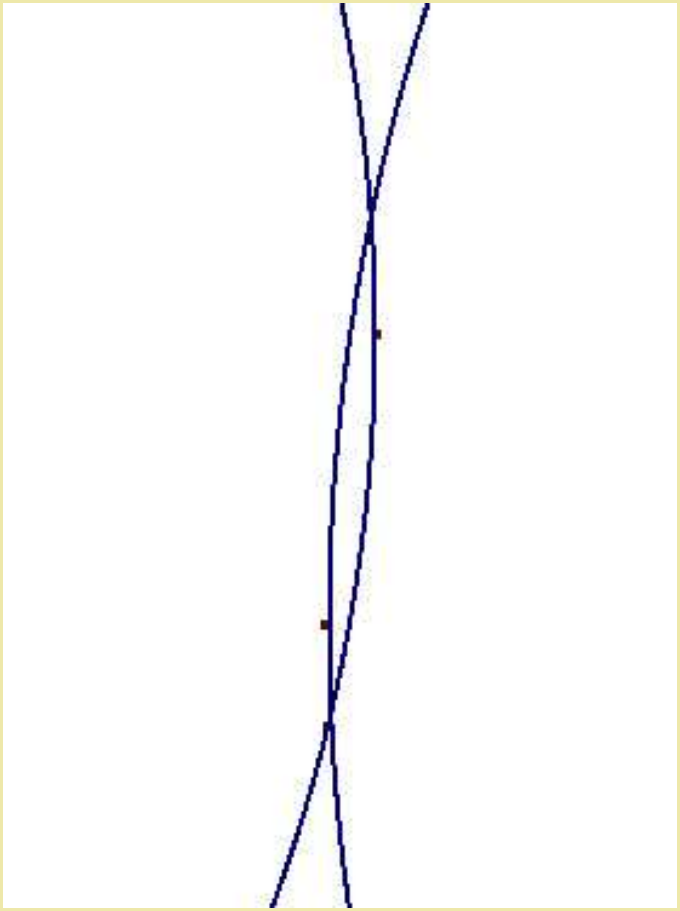


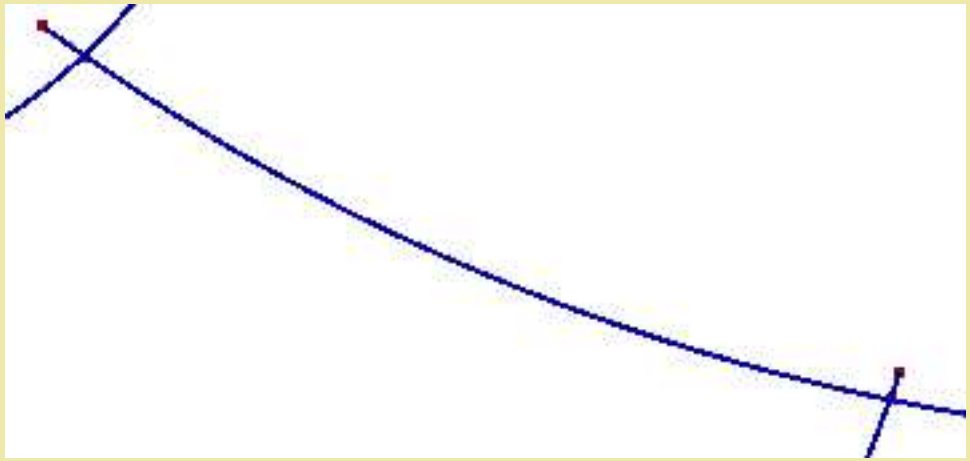
Arrangement of Circular Arcs



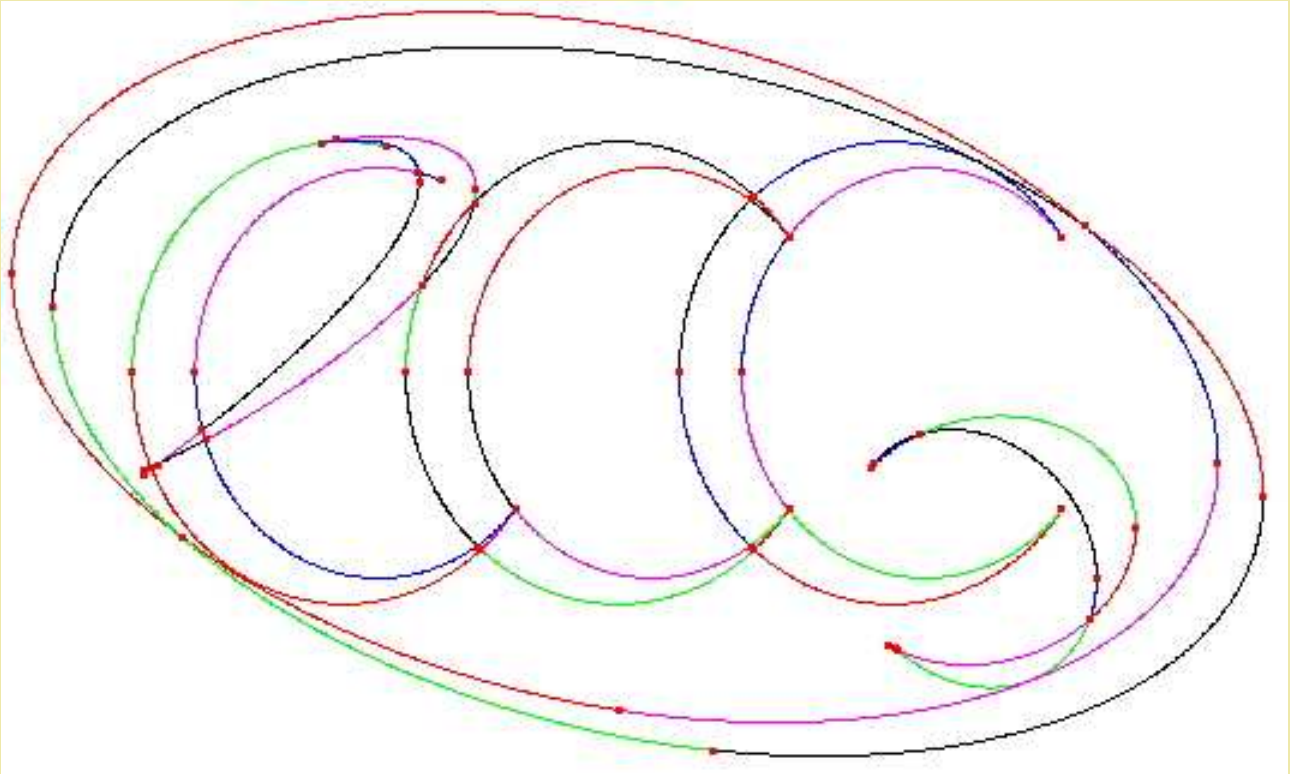
Arrangement of Circular Arcs

Details



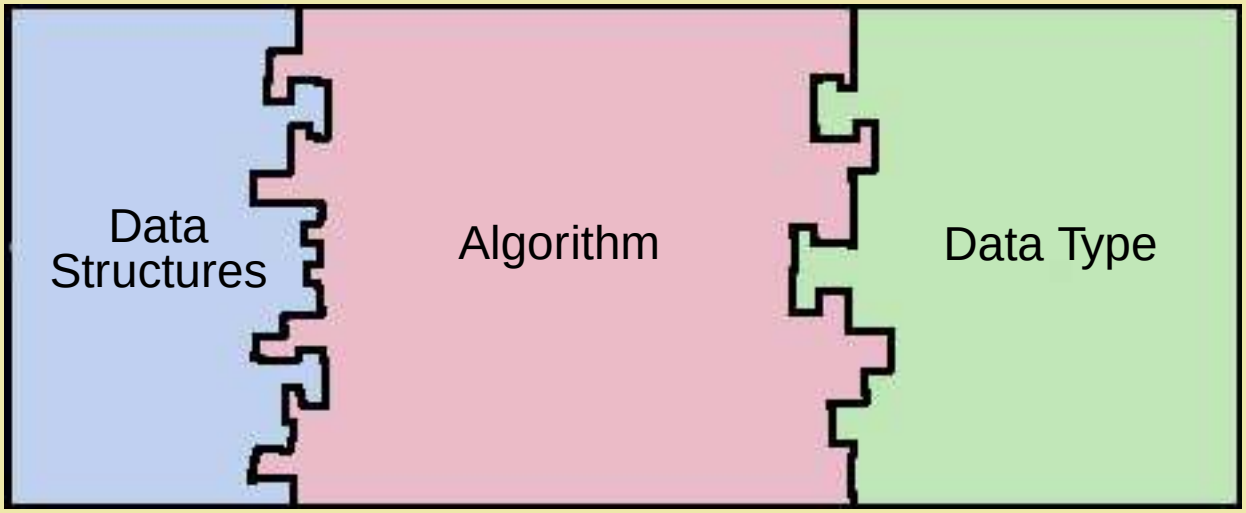


Arrangement of Elliptic Arcs



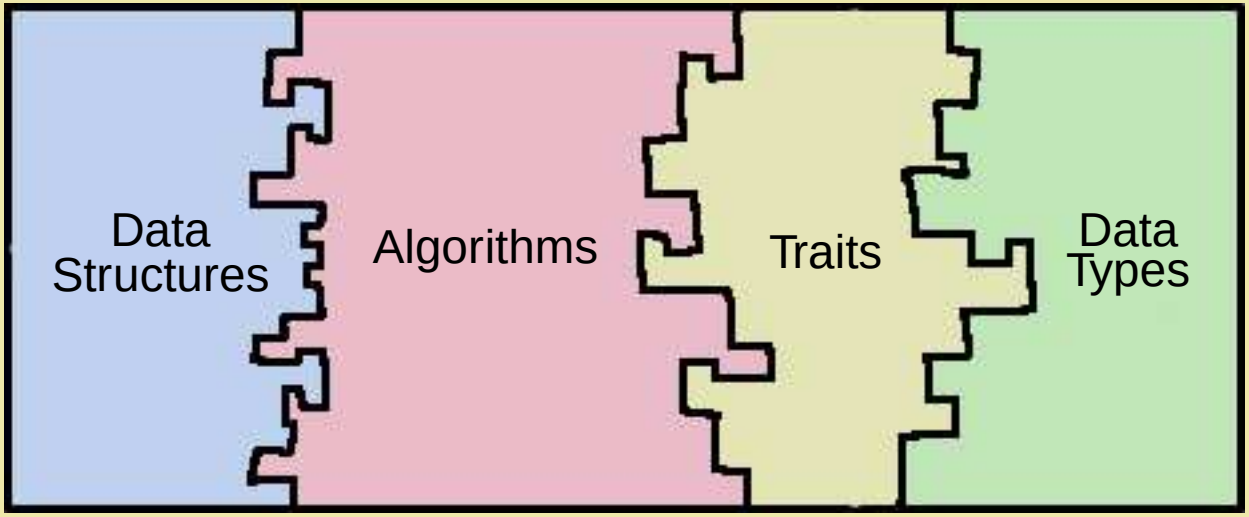
CGAL Introduction

Program anatomy



Library anatomy

Adaptors : Traits classes



Concepts, Models

in short

- Concepts define Interfaces and reside in the documentation

- Models provide Implementation and reside in the sources

The Geometric Primitives

A collection that contains among others

- Point
- Line
- Segment
- Circle
- Conic

The basic Geometric Data Structures

a collection that contains among others

- Polygons
- Half-edge data structures
- Topological maps
- Triangulations
- Multidimensional search trees

The Geometric Algorithms

are parametrized by

- The Data Structures
- A Traits class

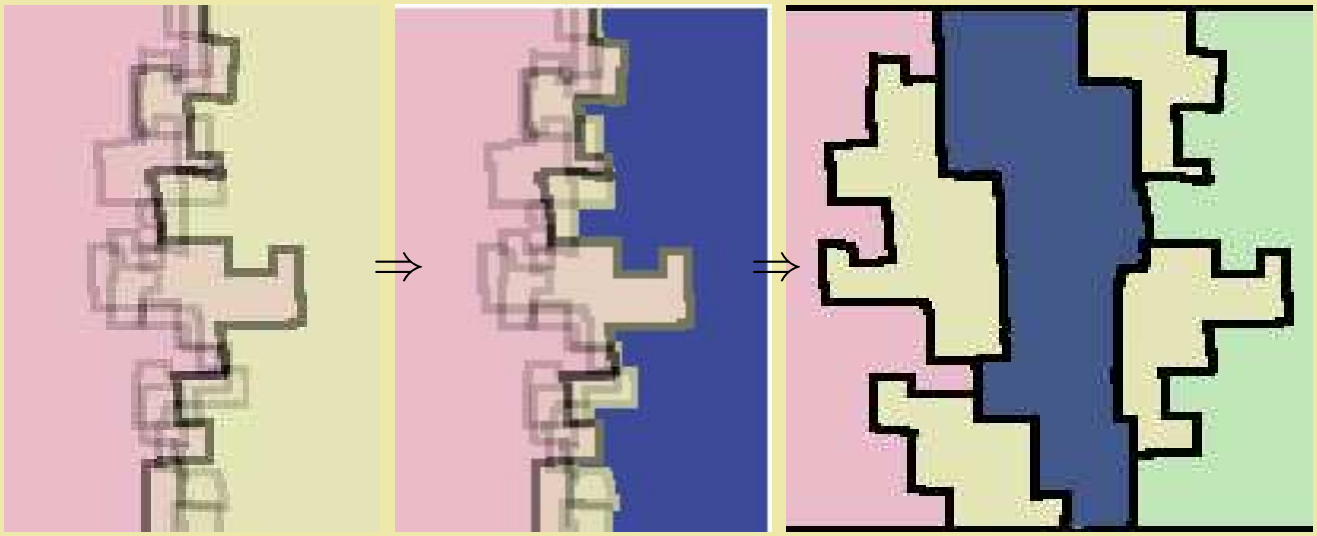
The Traits classes

define interface between

- The Data Structures
- Algorithm
- Primitives

The Kernel

factoring out common Traits functionality



The Geometric Kernel

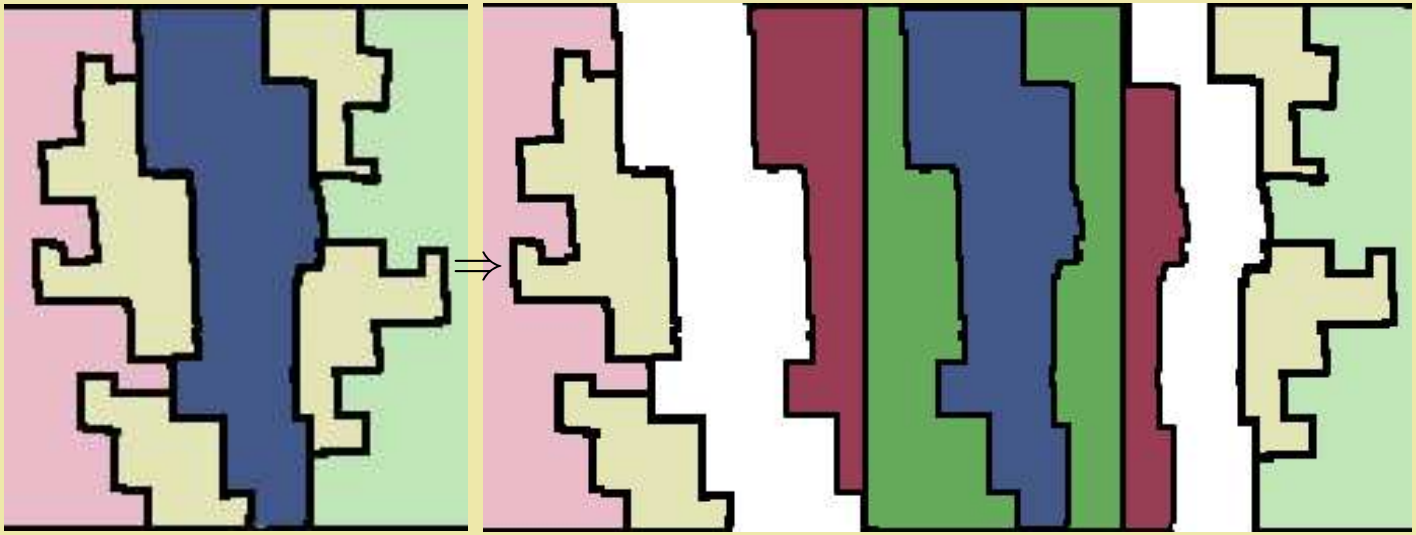
groups

- constant-size non-modifiable Geometric Primitive objects (Point, Line)
- operations on the above objects (ccw(), less_xy())

has models

- CGAL::Cartesian
- CGAL::Homogeneous

extending/exchanging the Kernel



The Curved Kernel

The Curved Kernel

is parametrized by

- A Linear Kernel (Circles, Conics, Points)
- An Algebraic Kernel (Algebraic Number Type, Equation Type)

extends/defines

- Conic
- Conic arcs
- Intersection, End Points of Conics

The Conic

is extended to

- Operate with Conic arc Endpoints

- Provide the implicit equation

The Conic arc

is defined by

- A supporting Conic
- A pair of Conic arc Endpoints

The Conic arc Endpoint

is defined by

- A pair of Conics that intersect on this point (implicit representation)

- A Point with coordinates of Algebraic NT (explicit representation)

The Algebraic Kernel

The Algebraic Kernel

is parametrized by

- A ring number type

defines

- Bivariate polynomials
- Algebraic numbers of degree up to 4

has model

- ECG::Synaps_kernel

Bivariate Polynomial

supports

- Sign at a pair of algebraic numbers
- Symbolic solve producing pairs of algebraic numbers
- Derivative wrt y

has models

- Synaps::BPoly_2_2
- Synaps::mpol

Algebraic Number Type

supports

- Three valued comparisons
- Sign

has models

- Synaps::root_of
- leda::real (with diamond operator)
- CORE::Expr

Algebraic Predicates

- *Sign* **compare**(*RootOf* , *RootOf*)
- *Sign* **sign**(*RootOf*)
- *Sign* **sign_at**(*BPoly* , *Pair*$<$ *RootOf* $>$)
- *Sequence*$<$ *Pair*$<$ *RootOf* $>>$ **solve**(*BPoly* , *BPoly*)

The Arrangement Traits

The Arrangement Traits

defines

- An x monotone curve
- Geometric Predicates on x monotone curves

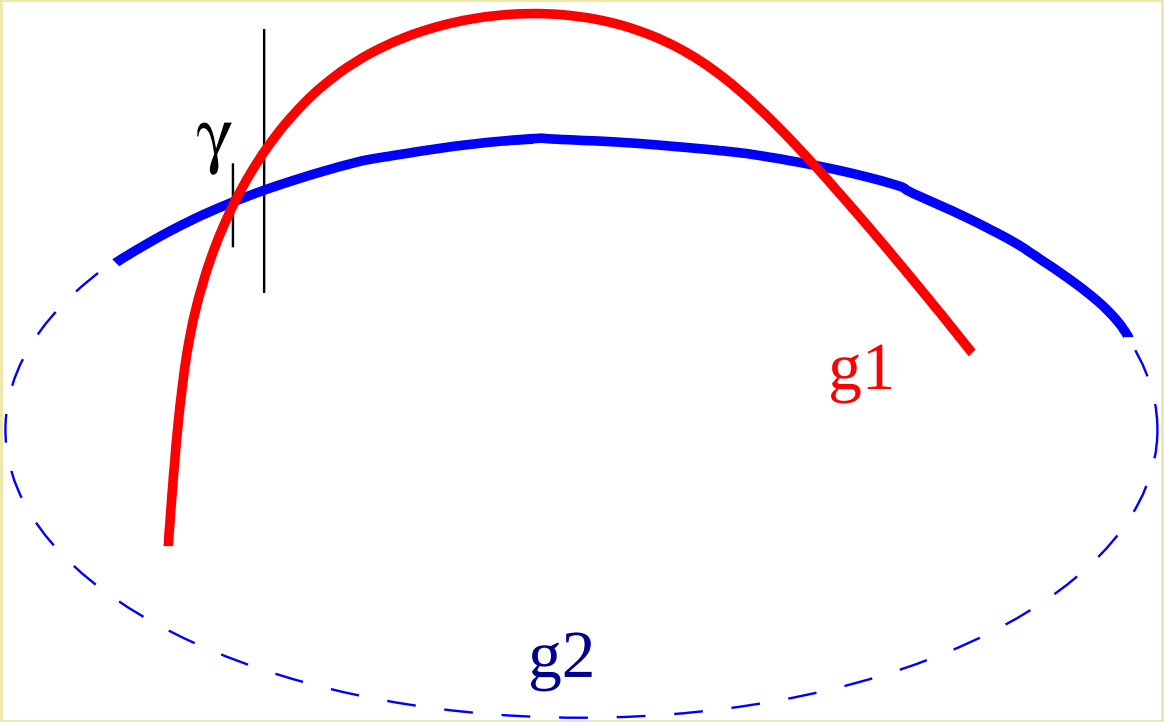
has model

- ECG::Conic_arc_traits
- CGAL::Arr_conic_traits_2

Geometric Predicates for Arrangement of curves

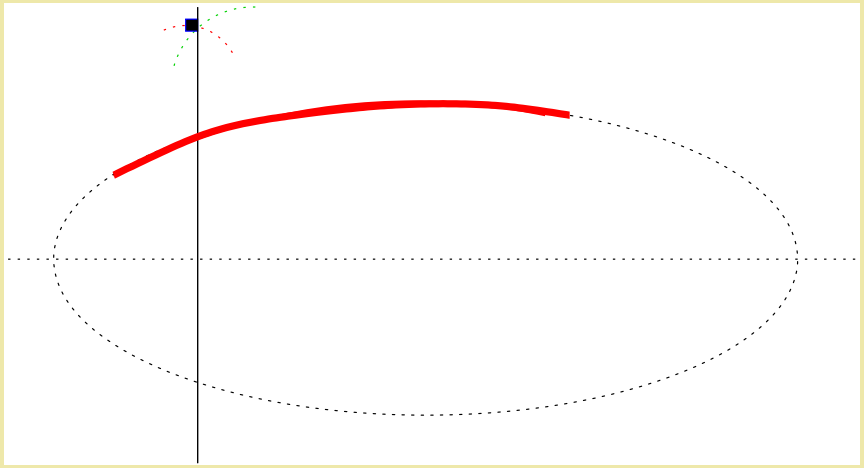
- Point comparisons in x, y and xy order
- Curve comparisons in y order
- Curve - Point comparisons in x and y order
- Curve - Curve intersection
- Curve test/make x monotone
- Curve split at a point

Nearest Intersection to the right



- Algebraic::solve
- Algebraic::compare

Point, Curve Compare y at x



- Algebraic::sign_at

- Algebraic::diff

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The End